MTH 310 HW 11 Solutions

April 23, 2016

Homework Problem 1

Prove that if $f(x) \in \mathbb{Z}_2[x], f(x^2) = f(x)^2$.
Answer. Let $f(x) = a_n x^n + ... + a_1 x + a_0$. Then $f(x)^2 = (a_n x^n + ... + a_1 x + a_0)^2 = (a_n^2 x^{2n} + ... + a_1^2 x^2 + a_0^2) + 2\sum_{i<j<n} a_i a_j x^{i+j} = a_n^2 x^{2n} + ... + a_1^2 x^2 + a_0^2 = a_n x^{2n} + ... + a_1 x^2 + a_0$ since if $k \in \mathbb{Z}_2 = \{0, 1\}, k^2 = k$.

Section 6.1, Problem 4

Is the set $J = \{\begin{bmatrix} 0 & 0 \\ 0 & r \end{bmatrix} | r \in \mathbb{R}\}$ an ideal of $M_{2x2}(\mathbb{R})$?
Answer. No. Consider the matrix L $= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \in J$ and $K = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Then $JK = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \notin J$.

1 Section 6.1, Problem 11

List the distinct principal ideals in $\mathbb{Z}_5$ and $\mathbb{Z}_9$
Answer. In $\mathbb{Z}_5$, the principal ideals are $(0), (1), (2), (3), (4)$. But all nonzero principal ideals are $\mathbb{Z}_5$. Therefore the distinct principal ideals are $(0), (1)$.

In $\mathbb{Z}_9$, the principal ideals are $(0), (1), (2), (3), (4), (5), (6), (7), (8)$. But 1 is in all of $(1), (2), (4), (5), (7), (8)$. However, $1 \notin (3)$ or (6). We also have $(3) = \{0, 3, 6\} = (6)$ so the distinct ideals are $(0), (1), (3)$.